How to deal with undesired non-monotonicity in machine learning and decison making?

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In many modelling problems, there exists a monotone relationship between one or more of the input variables and the output variable, although this may not always be fully the case in the observed input-output data due to data imperfections. Monotonicity is also a common property of evaluation and selection procedures. In contrast to a local property such as continuity, monotonicity is of a global nature and any violation of it is therefore simply unacceptable. We explore several problem settings where monotonicity matters, including fuzzy modelling, machine learning and decision making.

By far the most popular fuzzy modelling paradigm, despite its weak theoretical foundations, is the rule-based approach of Mamdani and Assilian. In numerous applied papers, authors innocently assume that given a fuzzy rule base that appears monotone at the linguistic level, this will be the case for the generated input-output mapping as well. Unfortunately, this assumption is false, and we will show how to counter it. Moreover, we will show that an implication-based interpretation, accompanied with a cumulative approach based on at-least and/or at-most quantifiers, might be a much more reasonable alternative.

Next, we deal with a particular type of classification problem, in which there exists a linear ordering on the label set (as in ordinal regression) as well as on the domain of each of the features. Moreover, there exists a monotone relationship between the features and the class labels. Such problems of monotone classification typically arise in a multi-criteria evaluation setting. When learning such a model from a data set, we are confronted with data impurity in the form of reversed preference. We present the Ordinal Stochastic Dominance Learner framework, which permits to build various instance-based algorithms able to process such data. Moreover, we explain how reversed preference can be eliminated by relating this problem to the maximum independent set problem and solving it efficiently using flow network algorithms.

Finally, we explore a pairwise preference setting where each stakeholder expresses his/her preferences in the shape of a reciprocal relation that is monotone w.r.t. a linear order on the set of alternatives. The goal is to come up with an overall monotone reciprocal relation reflecting `best' the opinions. We formulate the problem as an optimization problem, where the aggregated linear order is that for which the implied stochastic monotonicity conditions are closest to being satisfied by the distribution of the input monotone reciprocal relations. A monotone reciprocal relation is then easily found on the basis of the (possibly) constructed stochastically monotone reciprocal distributional relation. Interesting links with social choice will be pointed out.

As will be explained, central to the above three settings is the cumulative approach, which matches nicely with the monotonicity requirement.

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